

# Photoexcited-carrier-induced refractive index change in small bandgap semiconductors

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Using accurate band structures of InAs, InSb, and two  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  alloys, we calculate the change in refractive index caused by the photoexcited electrons and holes. The effects of both free-carrier absorption (FCA) and one-photon absorption are considered. We find that the change in refractive index varies nonlinearly with the density of photoexcited carriers and that the generally neglected FCA contribution is significant in InAs, owing to its weak spin-orbit coupling. © 2006 Optical Society of America  
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## 1. INTRODUCTION

Semiconductor materials such as InSb, InAs, and HgCdTe are used in optoelectronic applications including quantum-cascade lasers, terahertz generation, and saturable absorbers.<sup>1,2</sup> These applications involve generation of high levels of charge carriers that render it important to understand the effect of photoexcited carriers on the material optical properties. In addition, high-intensity light propagation in semiconductors can be strongly affected by the generation of a high density of photoexcited carriers. The incident light creates electron-hole pairs, some of which recombine through a variety of mechanisms including the Auger, radiative, and Shockley-Read-Hall processes. Not all electron-hole pairs will recombine before the entry of the next stream of photons, and the excess excited carriers will alter the photon absorption both above and below the fundamental gap. As a result, the refractive index changes from its value at equilibrium. Although there are a number of studies to accurately calculate the absorption and carrier relaxation in semiconductors, only simplified models that use effective mass band structures, the Boltzmann distribution for the carriers, and one-photon absorption (OPA) across the bandgap are currently being employed in the evaluation of refractive index changes.<sup>3-8</sup> These calculations predict that the change in refractive index varies linearly with the carrier density and increases with lattice temperature. The contribution from free-carrier absorption (FCA) is considered only within the Drude approximation.<sup>9</sup>

In this paper, we use full band structures and Fermi-Dirac (FD) statistics for photoexcited carriers to systematically calculate the change in refractive index arising from both OPA and FCA in four direct bandgap semiconductors: two having bandgaps less than 0.21 eV (InSb and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$ ) and two having a bandgap of approximately 0.35 eV (InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$ ). The results from this accurate calculation differ from the existing re-

sults not only in magnitude, but also in the trend. The calculated change in refractive index varies highly nonlinearly with the excess carrier density in the two smaller bandgap materials but varies almost linearly in the two larger bandgap materials and has a considerable contribution from FCA in InAs, where the spin-orbit (SO) splitting is small.

## 2. FORMALISM

The photoexcited carriers in the conduction and valence bands modify the absorption spectrum, which results in a change in the refractive index through the Kramers-Kronig relation between the real part of the refractive index  $n_1$  and the absorption coefficient  $\alpha(\omega)$ :

$$n_1(\omega) - 1 = \frac{c}{\pi} P \int_0^\infty \frac{\alpha(\omega') d\omega'}{\omega'^2 - \omega^2}, \quad (1)$$

where  $c$  is the speed of light,  $\omega$  is the angular frequency, and  $P$  indicates the principle value of the integral. The refractive index change  $\Delta n_1$  arising from the excess carriers can be calculated from<sup>10</sup>

$$\Delta n_1(\omega) = \frac{c}{\pi} P \int_0^\infty \frac{\alpha(\omega', \Delta N) - \alpha(\omega', 0)}{\omega'^2 - \omega^2} d\omega', \quad (2)$$

where  $\alpha(\omega', \Delta N)$  and  $\alpha(\omega', 0)$  are the absorption coefficients at angular frequency of  $\omega'$  in the presence of photoexcited electron-hole pairs with the density of  $\Delta N$  and in equilibrium, respectively.

The absorption coefficient is related to the imaginary part of the dielectric function,  $\epsilon_2(\omega)$ , via  $\alpha(\omega) = [\omega \epsilon_2(\omega)]/[n_1(\omega)c]$ . For a small change in refractive index,  $n_1(\omega)$  in this relation can be replaced by  $n_1^0(\omega)$ , which is the refractive index for the system in equilibrium. The absorption change can then be written in terms of the change in the dielectric function,

$$\alpha(\omega, \Delta N) - \alpha_0(\omega) = \frac{\omega}{n_1^0(\omega)c} [\epsilon_2(\omega, \Delta N) - \epsilon_2(\omega, 0)], \quad (3)$$

where  $\epsilon_2(\omega, 0)$  and  $\epsilon_2(\omega, \Delta N)$  are the imaginary part of the dielectric function for the system in equilibrium and in the presence of additional electron–hole pairs with density  $\Delta N$ , respectively.

Among many absorption processes, the FCA and OPA are most sensitive to the carrier occupation in a small bandgap material. The OPA excites an electron from the valence band into the conduction band when the photon has energy greater than the bandgap. To achieve this interband transition, the initial state in the valence band must be occupied and the final state in the conduction band must be vacant. The photoexcited carriers change the occupation in both conduction and valence bands when compared with the system in equilibrium and thus modify the OPA. When the energy of a photon is not large enough to enable valence-band to conduction-band transitions, the carriers can still absorb the photon through intraband transitions (with the help of phonons) and intervalence-band transitions, commonly known as FCA. Since both these processes are proportional to the number of carriers, the photoexcited carriers will modify the FCA spectrum as well.

We calculate the change in the dielectric function arising from the OPA and FCA by electrons and holes using accurate band structures and wave functions from a long-range tight-binding Hamiltonian.<sup>11</sup> The OPA contribution to  $\epsilon_2(\omega, \Delta N)$  can be calculated from

$$\begin{aligned} \epsilon_2(\omega, \Delta N) &= \frac{4\pi^2 e^2}{3m_0^2 \omega^2} \sum_{nm\mathbf{k}} |\mathbf{p}_{nm}(\mathbf{k})|^2 f(E_{m\mathbf{k}}) [1 - f(E_{n\mathbf{k}})] \\ &\times \delta(E_{n\mathbf{k}} - E_{m\mathbf{k}} - \hbar\omega), \end{aligned} \quad (4)$$

where  $e$  is the electron charge;  $m_0$  is the free-electron mass;  $n$  and  $m$  are the conduction- and valence-band indices, respectively;  $E_{n\mathbf{k}}$  and  $E_{m\mathbf{k}}$  are the corresponding band energies; and  $p_{nm}$  are the interband momentum matrix elements.  $f$  is the charge-carrier distribution function,

$$\begin{aligned} f(E_{m\mathbf{k}}) &= [1 + e^{\beta(E_{m\mathbf{k}} - \mu_h)}]^{-1}, \\ f(E_{n\mathbf{k}}) &= [1 + e^{\beta(E_{n\mathbf{k}} - \mu_e)}]^{-1}, \end{aligned} \quad (5)$$

where  $\mu_e$  and  $\mu_h$  are the Fermi levels for electrons and holes, respectively, which are determined from the density of photoexcited electron–hole pairs,  $\Delta N$ . When  $\Delta N = 0$ ,  $\mu_e$  and  $\mu_h$ , are equal.

The FCA contribution to  $\epsilon_2(\omega, \Delta N)$  includes two parts. The first is that electrons in the conduction bands (or holes in the valence bands) absorb a longitudinal-optical photon and make a transition to a higher-energy state within the same band. This contribution is calculated from

$$\epsilon_2(\omega, \Delta N) = \frac{8\pi^3 e^4 \omega_{\text{LO}}}{\hbar \omega^3 m_0^2 V} \left( \frac{1}{\kappa_\infty} - \frac{1}{\kappa_0} \right) \sum_{n\mathbf{k}} f(E_{n\mathbf{k}}) R_{n\mathbf{k}}, \quad (6)$$

$$\begin{aligned} R_{n\mathbf{k}} &= \sum_{n\mathbf{q}\pm} \left( N_{\text{LO}} + \frac{1}{2} \mp \frac{1}{2} \right) \left| \left\langle n\mathbf{k} \pm \mathbf{q} \left| \frac{e^{i\mathbf{q}\cdot\mathbf{r}}}{q} \right| n\mathbf{k} \right\rangle \right. \\ &\times \left( \left\langle n\mathbf{k} \left| \sum_j \mathbf{e}_j \cdot \mathbf{p} \right| n\mathbf{k} \right\rangle \right. \\ &\left. \left. - \left\langle n\mathbf{k} + \mathbf{q} \left| \sum_j \mathbf{e}_j \cdot \mathbf{p} \right| n\mathbf{k} + \mathbf{q} \right\rangle \right) \right|, \end{aligned} \quad (7)$$

where  $n$  is the band index,  $\hbar\omega_{\text{LO}}$  is the LO phonon energy, and  $N_{\text{LO}}$  is the number of LO phonons that can be obtained from the Bose–Einstein distribution function. We have used the Fröhlich Hamiltonian to describe the coupling between the electrons and the LO phonons. The second part arises from photon absorption by holes, without a phonon, making a direct transition between the heavy-hole and light-hole valence bands. This contribution is calculated from

$$\begin{aligned} \epsilon_2(\omega, \Delta N) &= \frac{4\pi^2 e^2}{3m_0^2 \omega^2} \sum_{mm'\mathbf{k}} |\mathbf{p}_{mm'}(\mathbf{k})|^2 f(E_{m\mathbf{k}}) [1 - f(E_{m'\mathbf{k}})] \\ &\times \delta(E_{m'\mathbf{k}} - E_{m\mathbf{k}} - \hbar\omega), \end{aligned} \quad (8)$$

where  $m$  and  $m'$  are valence-band indices.

It should be noted from Eqs. (2) and (3) that the dielectric function needs to be computed over a very large frequency range even if  $\Delta n_1$  is evaluated only near the band-edge frequencies. In addition, a careful numerical evaluation of the integral in Eq. (2) is required because of the singularity in the integrand.

### 3. RESULTS AND DISCUSSION

To test the accuracy of the band structures and dipole matrix elements used in our calculations, we have computed the OPA coefficients in InSb and InAs and found them to be in excellent agreement with the measured values. The dielectric function and the change in the refractive index are then calculated for the two materials InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$  with bandgap  $E_g$  of 0.35 eV and the two smaller bandgap materials in Sb at  $E_g=0.173$  eV and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$  with  $E_g=0.206$  eV. The spectra of  $n_1^0(\omega)$  used in our calculations are from the literature.<sup>12–15</sup>

Figure 1 shows the calculated change in refractive index as a function of wavelength due to the FCA and OPA in InSb at two densities of photoexcited electron–hole pairs— $\Delta N=10^{16}$  and  $10^{17}$   $\text{cm}^{-3}$ . We see that the FCA contribution depends weakly on the wavelength. However, the OPA contribution shows a steep change near the bandgap. This variation can be easily understood from Eq. (2). In the presence of photoexcited carriers, the band-edge states are filled, and the joint density of states of electrons and holes available for absorption decreases and the OPA is reduced. Thus the numerator of the integrand in Eq. (2) is either zero (for  $\omega' < \omega_g$ ) or negative (for  $\omega' > \omega_g$ ) with  $\omega_g$  being the bandgap frequency. When the frequency  $\omega$  crosses  $\omega_g$ , the denominator of the integrand changes sign and consequently  $\Delta n_1(\omega)$  changes sign.

The calculated value of  $\Delta n_1$  at  $\lambda=9.5$   $\mu\text{m}$  is shown in Fig. 2 as a function of photoexcited carrier density in InSb and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$ . We see that in both materials the

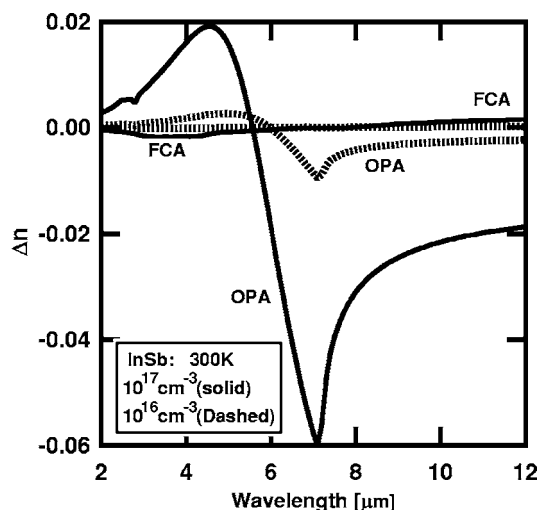


Fig. 1. Refractive index change due to the FCA and OPA as a function of the photon wavelength in InSb with  $\Delta N=10^{16}$  and  $10^{17} \text{ cm}^{-3}$  at room temperature.

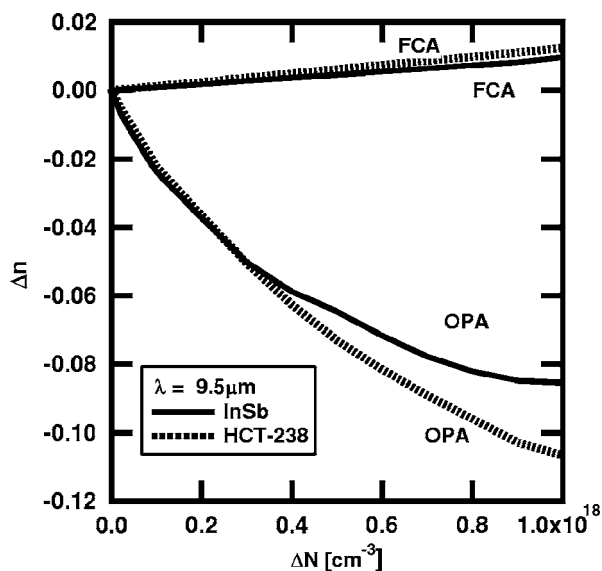


Fig. 2. Refractive index change at  $\lambda=9.5 \mu\text{m}$  due to the FCA and OPA as a function of  $\Delta N$  in InSb and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$  (HCT) at room temperature.

OPA contribution dominates over the FCA contribution to the  $\Delta n_1$ . Interestingly, the FCA contribution is positive, which is opposite of the OPA contribution at this wavelength. Similarly calculated results at  $\lambda=4.8 \mu\text{m}$  for the larger bandgap materials—InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$ —are plotted in Fig. 3. In this case, both FCA and OPA reduce the refractive index. Although both materials have the same bandgap, in the HgCdTe alloy the dominant contribution is from the OPA, whereas in InAs both FCA and OPA have comparable contributions. To understand the different levels of contribution of FCA to the refractive index change, we plot the FCA cross section  $\sigma$  as a function of wavelength in Fig. 4 for these two materials. The numerator of the integrand in Eq. (2) for the FCA process is proportional to this cross section, and the FCA contribution to  $\Delta n_1$  at a photon wavelength  $\lambda$  is roughly proportional to the difference in the areas below

and above that  $\lambda$  in Fig. 4. We see that the absorption for wavelengths below  $\lambda=4.8 \mu\text{m}$  is negligible in InAs, but is significant in the HgCdTe alloy. Owing to a larger cancellation, the FCA contribution is smaller in the HgCdTe alloy. We explore further to understand the origin of the difference in the absorption spectrum of Fig. 4. In our calculations we found that most of the FCA arises from the transitions of holes from the heavy-hole to light-hole bands. Hence the different FCA spectra can be understood from the valance-band structures in the two materials. We plot in Fig. 5 the energy difference between the heavy-hole and the light-hole valance bands as a function of wave vector  $\mathbf{k}$  in symmetry directions for InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$ . In both cases the energy difference in-

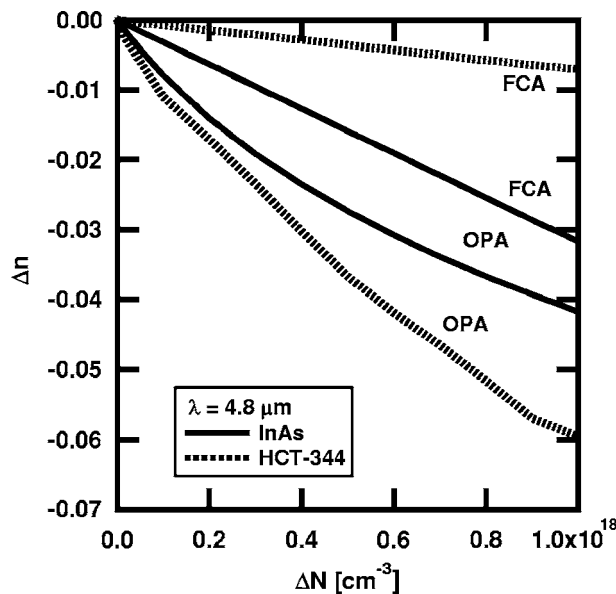


Fig. 3. Refractive index change at  $\lambda=4.8 \mu\text{m}$  due to the FCA and OPA as a function of  $\Delta N$  in InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$  (HCT) at room temperature.

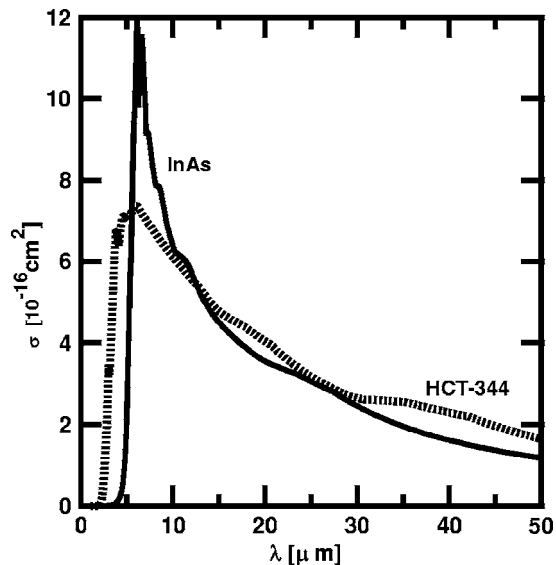


Fig. 4. Cross section of the FCA  $\sigma$  as a function of the photon wavelength in InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$  (HCT) at room temperature.

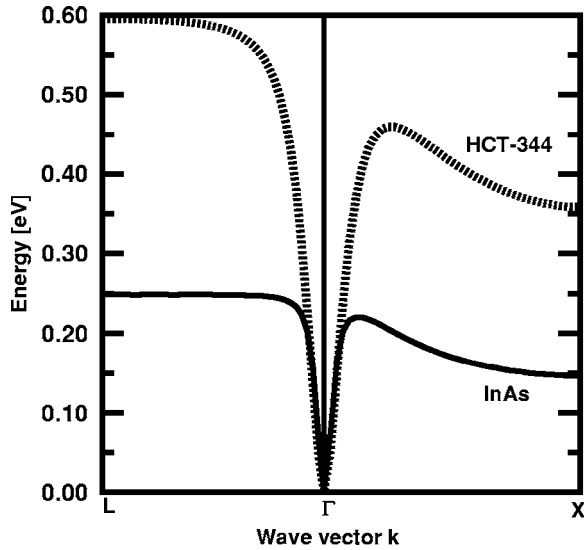


Fig. 5. Energy difference between the heavy-hole and light-hole valence bands as a function of wave vector for InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$  (HCT) at room temperature.

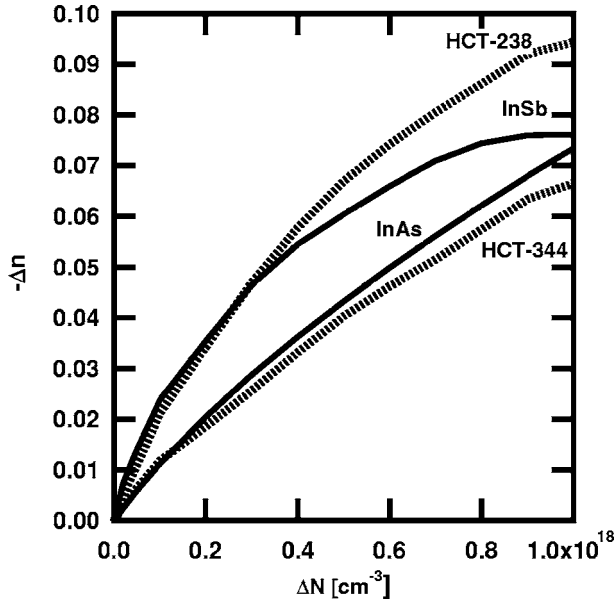


Fig. 6. Total change in the refractive index  $\Delta n_1$  at  $\lambda=4.8 \mu\text{m}$  for InSb and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$  (HCT-238) and at  $\lambda=9.5 \mu\text{m}$  for InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$  (HCT-344) as a function of  $\Delta N$  at room temperature.

**Table 1. Fit Parameters to Obtain the Calculated Change in Refractive Index at 300 K**

Parameter	InAs	$\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ ( $x=0.344$ )	InSb	$\text{Hg}_{1-x}\text{Cd}_x\text{Te}$ ( $x=0.238$ )
$a$	0.13067	0.14536	0.077854	0.12091
$b$	$8.1952 \times 10^{-19}$	$7.079 \times 10^{-19}$	$2.9875 \times 10^{-18}$	$1.6804 \times 10^{-18}$

creases rapidly at small values of  $\mathbf{k}$ , but saturates at 0.25 eV as the wave vector increases further in InAs. This small saturation value can be attributed to the weak SO coupling (0.38 eV) in InAs, which repels the light-hole valence band closer to the heavy-hole valence band. Absorp-

tion of photon with a larger energy than this saturation value is not possible. In the HgCdTe alloy, however, a larger SO coupling (0.95 eV) allows the light-hole band to move farther away from the heavy-hole band, resulting in absorption at shorter wavelengths. Hence the relative contribution from OPA and FCA depend critically on the subtle details of the band structures.

The sum of the FCA- and OPA-induced change in refractive index, calculated at  $4.8 \mu\text{m}$  for the large gap materials and at  $9.5 \mu\text{m}$  for the small gap materials, is shown in Fig. 6 as a function of the density of photoexcited electron-hole pairs. Clearly the variation is nonlinear for the two smaller bandgap materials. An accurate fit to the total change indicated that variation is nonlinear even for the larger bandgap materials. Excellent fits to all four materials are obtained by using a functional form  $\Delta n_1 = a(1 - e^{-b\Delta N})$ . Only for small  $\Delta N (< 10^{16} \text{ cm}^{-3})$  does the refractive index change linearly with  $\Delta N$ . The fitted values of  $a$  and  $b$  are given in Table 1.

It is interesting to estimate the effect of temperature on the OPA-induced change in the refractive index. In our calculation with FD statistics and full band structures, the excess carrier-induced change in OPA is nearly temperature independent. However, for example, the bandgap in InSb changes from 0.173 eV at 300 K to 0.234 eV at 77 K. Since the contribution arising from OPA is nonzero only when  $\omega' > \omega_g$ , the denominator of the integrand in Eq. (2) is larger at  $T=77 \text{ K}$  than at 300 K, resulting in a smaller change in the index. This conclusion is in contradiction with previous studies,<sup>5,6</sup> which used simplified band structures and a Boltzmann distribution.

## 4. CONCLUSIONS

We have calculated the change in refractive index arising from both the OPA and the FCA using full band structures for four materials: InSb,  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$ , InAs, and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$ . We focus on the refractive index change at  $\lambda=9.5 \mu\text{m}$  for the first two materials that have bandgaps less than 0.21 eV and at  $\lambda=4.8 \mu\text{m}$  for the last two materials that have a bandgap of 0.35 eV. The calculations show that the FCA contribution is not negligible and is comparable to the OPA contribution in InAs that has a small SO splitting. The calculated dependence of the change in refractive index on the density of photoexcited electron-hole pairs is strongly nonlinear in InSb and  $\text{Hg}_{0.762}\text{Cd}_{0.238}\text{Te}$  and moderately nonlinear in InAs and  $\text{Hg}_{0.656}\text{Cd}_{0.344}\text{Te}$ . Furthermore, the change in refractive index for InSb due to the OPA is smaller at lower temperatures than at room temperature because of a larger bandgap at low temperatures.

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